OLIVER HEAVISIDE 1850 – 1925

by BALTHASA VAN DER POL
translated by L. J. DeFELICE

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At the beginning of this century, in Torquay on the southern coast of England, lived an old man who had spent most of his years secluded from the rest of the world. This recluse was deaf and all but abandoned by the world; whenever he left his house, he was teased by children as the local eccentric. His estrangement from others was such that, when he was quite sick and placed on a stretcher against his will to be taken to a hospital, he was apprehensive that he would be taken to a mental institution, and asked the village policeman to drive the hospital attendant away. Soon afterwards, in 1925 and in his 75th year, Oliver Heaviside died.

This self-made man never studied at a university or technical institution. I speak about Heaviside, not for historical reasons (electrical engineering is growing too quickly for us to spend much time dwelling on the past), but because his work is little known and yet is of such great importance to electrical communication engineering and its development.

Some – and perhaps the most important – articles by Heaviside never appeared in print. They were rejected by the editors of scientific and technical journals because of their excessive unreadability. Such editorial decisions are understandable when the articles that Heaviside did publish are considered. The order of the subject matter, the manner of treating the material, as well as the copious mathematical formalism, are so different from conventional scientific and technical literature that, without exaggeration, Oliver Heaviside's works are nearly unreadable, or they were at the time they were written.

Prominent people brought the unreadability of Heaviside's articles to his attention. In 1891 Lord Rayleigh complained about one of these articles in a letter to Heaviside:

> One [of the referees] says it is the most difficult he ever tried to read . . . As it is, I should fear that no one would take advantage of your work.[1]

Similar complaints came from Sir Oliver Lodge, Silvanus Thompson, Prof. FitzGerald and others.

Although Heaviside lived in seclusion, he often engaged in a lively correspondence with other physicists, such as Heinrich Hertz. 50 years ago, inspired by the theory of Maxwell, Hertz produced electromagnetic waves in a laboratory for the first time. As a byproduct, he discovered the photoelectric effect, and so laid the basis for radio and television. This eminent experimentalist and theoretician also complains in various letters to Heaviside about the unreadability of his articles:

> I find it so very difficult to follow your symbols and your very original mode of expressing yourself. (I quote from a letter dated 21 March, 1889.) [Reference 1, p. 238]

Another admonition appears in a letter written two months later:

> . . . So I felt obliged to give you warning that you are a little obscure for ordinary men. [Reference 1, p. 238]

The warnings become stronger in a letter from Hertz to Heaviside dated 31st December, 1890:

> . . . If you would only take a good form, a book of yours on the theory of electricity would have a great success in England and abroad. But I fear you have some pride in this, not to yield to the understanding of others. I think this is a false pride; you certainly are not aware how very difficult your papers are to understand to others and it is old wisdom that the many will expect you to come to them and not come up to you, be your merits ever so great. [Reference 1, p. 240]

The obscurity of Heaviside's written work was such that the publisher of the *Philosophical Magazine*, after learning that no one read what Heaviside wrote, discontinued the series of articles by Heaviside that had been published in this journal. Even the Royal Society, after publishing two of a series of three articles on 'Operators in physical mathematics,' informed Heaviside in a letter from the secretary:

> I am desired to return you the thanks of the Royal Society for your paper 'On Operators in Physical Mathematics, Part III,' and to inform you that the Committee on Papers, not thinking it expedient to publish it at present, have directed your manuscript to be deposited in the Archives of the Society. [Reference 1, p. 228]

Louis DeFelice is with the Department of Anatomy, Emory University, and the School of Electrical Engineering, Georgia Institute of Technology, Atlanta, Ga., USA

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In connection with this incident, the English mathematician, Prof. E.T. Whittaker, has written:

> 'There was a sort of tradition,' he said, 'that a Fellow of the Royal Society could print almost anything he liked in the Proceedings without being troubled by referees; but when Heaviside had published two papers on his symbolic methods, we felt that a line had to be drawn somewhere, so we put a stop to it.'[2]

Although Heaviside was understandably disheartened by the lack of appreciation for his work (for which he had his own writing to thank), he appeared never to have lost his English sense of humour. He writes:

> Experience has taught me that the refusal of a paper by any journal, for unconvincing conventional reasons, implies that the paper is unusually original and good. Fact! [Reference 1, p. 226]

or

> . . . one [of the Edinburgh school] said my work was 'a disgrace to the Royal Society,' to my great delight . . .[3]

In September 1938, Prof. Appleton told the International Radio Scientific Union that a full article of Heaviside's on what is now called the Heaviside layer was refused by *The Electrician.* This layer is the conducting layer high in the atmosphere that makes radiocommunication over large distances possible. Its existence is as important for the field of radio as iron in the earth is for all electrical engineering. Because the article was refused, what is known of Heaviside's work on this layer is confined to the following few sentences:

> There may possibly be a sufficiently conducting layer in the upper air. If so, the waves will, so to speak, catch on to it more or less. Then the guidance will be by the sea on one side and the upper layer on the other.[4]

Several years earlier (on the 7th May 1899), FitzGerald had written to Heaviside about Sir Oliver Lodge's work with the wireless telegraph and Marconi's experiments:

> Have you worked at the propagation of waves round a sphere? A case of this is troubling speculators as to the possibility of telegraphing by electromagnetic waves to America. It is evidently a question of diffraction and I think must be soluble. [Reference 1, pp. 247-248]

This problem was not taken up by Heaviside himself. It was originally studied by Poincaré, and later by mathematicians in England, including Nicholson, MacDonald, Love and Watson, and in Germany by Sommerfield and his students. Only recently can curves be drawn which describe how the field from the radio transmitter becomes weaker with distance under the influence of both absorption by the finite conduction of the ground, and by deflection around the curvature of the earth.

The unclear manner in which Heaviside wrote is the reason one rarely encounters references to his work in the literature. Is it not unusual that a writer, in the middle of a long mathematical argument about vector equations or about the penetration of electromagnetic fields in metals, suddenly enlarges on the slowing of his work due to a bout of influenza, or the uselessness of the study of Latin or Greek for physicists and engineers? (' . . . and in fact many people think it is about time the dead languages were buried.')[5]

Apart from these expressions, e.g. the sharp attacks on the use of quarternions in the theory of Maxwell, the writings of Oliver Heaviside present other difficulties to the reader. The practical engineer will have great difficulties in following the mathematical development. His mathematical course work will not help him through the original formalism which Heaviside uses. Yet it was Heaviside who demonstrated how to realise the long-distance cable telephone.

A mathematician would fare no better and will be annoyed with the horrible thing he sees. The rigorous mathematician familiar with Weierstrass and Kronecker will be dumbfounded at differential quotients of fractional order and of the first, second, third and even higher derivatives of a discontinous function. His horror will culminate at divergent power series from which calmly, and not infrequently, all even terms are thrown away. In connection with these divergent power series, Heaviside states:

> Mathematicians in general are, I find, exceedingly conservative and prejudiced. Nevertheless, I am confident of a great future for the practical use of divergent series, as well as for the generalised analysis which connects them with the convergent ones because both these matters are concerned in the operational treatment of physical differential equations. [Reference 3, p.287]

The rigorous mathematician will also be annoyed with the manner in which Heaviside solves his differential equations. He replaces classical methods with a short operational method in which, in the most kind-hearted way, discontinuous functions are differentiated, and strange, ill-behaved functions (whose existence is still not agreed on) are introduced.

Consider the function that is zero everywhere except at the origin where it becomes infinite. It appears in the literature well before Heaviside (for example, in Kirchhoff and Helmholtz, and, in my opinion, before them in Laplace). Heaviside writes:

> When mathematicians come to an infinity they are nonplussed and hedge around it. They would, for example, stick at the three sharp corners in the function above used, which involve discontinuity in the slope, or infinite curvature.[6]

and

> One shape of the function is just as easily conceived as another, and we are not limited to the angelically perfect function which is finite and continuous itself, and has all its derivatives, finite and continuous. [Reference 6, pp. 112-113]

Dirac (who perhaps derived it from Heaviside) has adapted this function into an important tool of wave mechanics. The pure mathematician (e.g. Hadamard or G. N. Watson), led to this function by physical means, either does not recognise the function or simply dismisses it. Heaviside, with sharp physical intuition, brings it in everywhere. His theory of the propagation of signals along cables uses the function in a way which gives correct results, but where the logical steps are anything but justified. No wonder that serious opposition to the computation methods of Heaviside developed in the mathematical community. An unfortunate estrangement of Heaviside and English mathematicians resulted, which led Heaviside to such biting expressions as:

> Even Cambridge mathematicians deserve justice. [Reference 6, p. 10]

and

> Whether good mathematicians, when they die, go to Cambridge, I do not know. [Reference 3, p. 175]

In this conflict both the mathematicians and the 'mathematical physicist' were right. The physicist who does not have the time or capacity to follow the proofs owes to mathematicians the modern, rigorous way of handling these problems, and obtains full guarantee for the usefulness of the results. A mathematician asks too much, if he insists that the physicist or engineer, before Fourier analysing an oscillogram of an alternating current, must first prove that the function is absolutely integrable and has a finite number of discontinuities.

The degree of mathematical rigour needed by the scientist or engineer is dependent on his physical insight. In the solution of theoretical physical problems, scientific insight into the phenomenon takes the place of rigour to a certain degree.

The introduction written by Lord Rayleigh in his standard work on 'Sound' is worth reading in this regard; a similar remark by Heaviside about the application of mathematics to physical and technical problems is interesting:

> The best result of mathematics is to do without it. [Reference 6, p. 7]

Poincaré first shows that some of the approximations used in the astronomy of planets were the first terms of asymptotic series, for which convergence was not rigorously demonstrated. Today, wave mechanics is in the pre-Poincaré state, which, from a historical perspective, need not cause concern. Another example is the work of the astronomer G. W. Hill, who calculated the movement of the perigee of the moon by introducing for the first time an infinite determinant; it was years before convergence of his procedure could be shown. I agree entirely with Prof. Kramers about rigour in mathematical physics: he would place the burden of proof of the incorrectness of the methods on the shoulders of the mathematicians. Heaviside often emphasised that the most beautiful mathematical problems, and the most interesting mathematical functions are frequently borrowed from scientific or

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technical questions. Consider the beautiful work that has come from potential equations and wave equations in three or more dimensions, and the well known theorem on the stability of the roots of algebraic equations that Hurwitz has derived in pursuit of unwanted vibrations caused by a large machine.

Undoubtedly, a middle ground can be found between the physicist and the mathematician, which should lie closer to the view of the mathematician than it did when Heaviside wrote: 'Mathematics is an experimental science.' [Reference 6, p.1]. Thanks to the better mathematical training of young physicists and engineers, this middle ground can now be found more easily. The time is past when Landau called pure mathematics 'Mathematik,' but scientific mathematics 'Belletristik,' [fiction] and technical mathematics 'Schmieröl,' [lubricating oil] and when a British mathematician could say: 'Bessel functions are beautiful functions in spite of their many applications.' [No source cited]

Heaviside is especially unique in the mathematical ways in which he handled physical problems. He continually insisted on the humanisation of engineering mathematics and presented results in the most simple way without too much 'Circumbendibus' [beating around the bush]. The term 'simple' has to be understood as relating to current physical notions. ('It seems to me that the demonstration I have poked fun at is typical of a lot of work made up by the brain-torturers who write books for young people and college students who are going to be senior Wranglers, perhaps. The best of all proofs is to set out the fact descriptively, so

'is unique in the mathematical ways in which he handled problems'

that it can be seen to be a fact.') [Reference 3, p.140]. Heaviside did not like the form in which mathematical results are frequently presented. Not uncommonly, a mathematical proposition is discovered along intuitive experimental lines; only later is a rigorous proof constructed. In mathematical literature, the proof is often given in form in which, according to Heaviside, 'the mathematicians pretend they knew all about it before they began.'

> But in every case there is a Providence in the form of an equivalent divergent formula, which is exactly suitable just when the convergent formula is of impossible utility practically. And the Providence is so good as to arrange matters secretly so that there is an overlapping region in which either formula may be employed, so that we may test that the one curve joins on properly to the other. It is wonderful that things should work out in this way. Logic has nothing to do with it, either with the fact, its discovery, or its use. At the same time, it must be said that a sufficiently profound study of the subject would be ultimately led to the logic of its laws as a final result. What I do strongly object to is the idea that the logic should come first, or else you prove nothing. Yet perhaps the majority of academical mathematical works are written under this idea. In reality the logic is the very last thing, and that is not final . . . And no doubt the logic of it all will have to be found out experimentally. And then, finally, I suppose 'rigorous' mathematicians will put the logic at the beginning, and pretend they knew all about it before they began. [Reference 3, p.370]

After this limited view of the intuitive manner in which Heaviside worked, and emphasising that my lecture is not a historical unravelling, but rather addresses the importance of Heaviside's work, I now turn to some theorems developed by Heaviside on such simple things as the functions e^x, $\sin x$, and $\cos x$, for which he has given new asymptotic series [Reference 6, p.466, and Reference 3, p.198]. Only recently has the correctness of the expansion of the exponential function been demonstrated rigorously. He also applies his concept of a 'generalised exponential' to Bessel functions,

and thus generates a new perspective of electromagnetic waves, which is not yet completely understood.

In addition, the fundamental understanding of Fourier series resulting from Heaviside's work merits the attention of mathematicians as well as physicists and engineers. This development derives from his considerations of disturbances which propagate along electrical cables provided with terminal impedances ('It illustrates a property of the wonderful function e^x.') [Reference 6, p.164 also pp. 160-165]. As far as I know, the enlargement of the Fourier series concept, as derived from an engineering problem, cannot be found in mathematical literature. Heaviside's operational calculus is of the greatest importance for modern wireless, as well as cable, communication. These mathematical methods, introduced and used by Heaviside in an experimental manner, are inspired by technical problems.

'the method lay dormant because of the manner of Heaviside's writing'

This method has been and still is the subject of much discussion in engineering literature. Some good books (but also many poor ones) exist on the subject. A poor book on the operational calculus should not be criticised too harshly. Even rigorous mathematicians have not been completely successful in justifying the Heaviside method, which continues to bring numerous mathematical relations to light. Evidence includes recent contributions from the pure mathematics. For many years, the operational calculus was completely neglected by European engineers, and ignored by mathematicians. Therefore, a statement from the English mathematician, Prof. E. T. Whittaker, which contrasts Heaviside's concept of mathematics with that of pure mathematicians, makes pleasant reading:

> Looking back on the controversy after thirty years, we should now place the Operational Calculus with Poincaré's discovery of automorphic functions and Ricci's discovery of the Tensor Calculus as the three most important mathematical advances of the last quarter of the nineteenth century. Applications, extensions and justifications of it constitute a considerable part of the mathematical activity of today. [Reference 2, p.216]

Happily, Prof. Elias, Bremekamp and others now refer to this matter in their lectures. The operational calculus involves a method of calculation, which is not only of great heuristic value in mathematics, but also gives insight into electrical coupling phenomena and electromagnetic waves; many consequences appear yet to be realised. For more than 20 years, the method lay dormant, largely because of the peculiar manner of Heaviside's writing. He himself writes, somewhat bitterly, about the small application in his time for the operational method:

> This is enough for the present about the operational treatment of definite integrals, which might go on forever. The above may help others on the way. But perhaps, like the fishes who were preached to by the saint, 'Much edified were they, but preferred the old way.' Very well, then there let them stay.' [Reference 3, p.291]

The operational method extends from mathematics to physics and engineering. With the help of this method, Heaviside had shown how to realise the long-distance cable telephone by 1878. In those days, the new theoretical considerations that did exist on the propagation of signals over cables were due to the work of Lord Kelvin, who concentrated on slowly varying phenomena, and neglected self induction. An expanded theory was developed by Heaviside, which led him quite early to the realisation that self induction would not only support signals over longer distances, but would noticeably diminish their distortion during propagation. He developed the conditions for a completely distortionless cable, and attained diminution in distortion in two ways: by increasing self induction per unit length, and by introducing self induction in the form of localised inductors:

> . . . viz. by the insertion of induction coils in the main circuit at regular intervals, say one per mile, according to circumstances. [Reference 3, p.291]. Another indirect way is this. Instead of trying to get a large uniformly spread inductance, try to get a large average inductance. Or, combine the two, and have large distributed inductance together with inductance in isolated lumps. This means the insertion of inductance coils at intervals in the main circuit. That is to say, just as the effect of uniform leakage may be imitated by leakage concentrated at distinct points, so we should try to imitate the inertial effect of uniform inductance by concentrating the inductance at distinct points. The more points the better, of course. [Reference 5, p.445]

Heaviside pointed out this result repeatedly, and insisted that experiments should be done that, by their very nature, could only take place through the General Post Office. He was opposed by the Postmaster General, Sir William Preece, who, through insufficient insight and acquaintance with the problem, fought his ideas and held up their introduction in England. People ridiculed the incorporation of inductors in cables, saying: 'It would be like making humps on a road to increase the speed of vehicles.' [Reference 1, p.254. Appleyard attributes this quotation to the physicist]. This opposition by the authorities gave Heaviside occasion to say: 'There seemed also to be an idea that official views, by virtue of their official nature, should not be controverted or criticised.' [Reference 2, p.208].

'Heaviside. . .was not on the best of terms with the official authorities'

Heaviside, through unfair and shortsighted treatment, was not on the best of terms with the official authorities. He writes in a discussion of the application of Heinrich Hertz's waves: ' . . . electromagnetic waves, not so long ago they were nowhere; now they

are everywhere, even in the Post Office'[7] and: 'But a dangerous and alarming official error has been pressed forward, even to the extent of experimentation with the public funds.' [Reference 6, p.5]. Although the UK failed to apply this discovery (owing to the misjudgment of its Postmaster General), the subject was taken up in America, and Heaviside refers to experiments done there by Pupin:

> Nothing particular has been done in Great Britain to carry out the writer's invention; but in America, some progress has been made by Dr. Pupin, who has described an experiment supporting its practicability; the length telephoned through was increased five times by inserting the coils. [Reference 3, p.345]

Because of these experiments, which Dr. Pupin conducted, one still hears 'Pupin inductors' spoken of, incorrectly, in the Netherlands.

Opposition from Sir William Preece did much injury to Heaviside throughout his life. People, including some Americans, later tried to soften the effect. Near the end of Heaviside's life, recognition of his merits began to come in England and America. A few months ago, I had the pleasure of seeing some old neglected and unpublished papers of Heaviside in the library of the Institution of Electrical Engineers in London. I found there a reprint of the American journal *Electrical World* dated 4th May 1918. The issue contained an article dedicated to the work of Heaviside, and naming him an honorary member of the American Institute of Electrical Engineers. The reprint contained marginal notes by Heaviside himself; among other things, I found these remarks:

> Officialdom was the enemy then and always . . . and it is admitted that America owes a great debt to me and it is said wants to make reparation to some degree, primarily for the great injury done to me by Pupin and his corporations. Then why don't they do it? It is not a matter of 'charity' nor is it one of 'business haggling' . . . My work has been absorbed.

Heaviside's small regard for official agencies is also evident from what he did with a photograph of venerable members of the Institution of Electrical Engineers:

> Giants at the back. Pigmies at the front. I gave it, framed and glazed too, away to a Newton Abbot furniture dealer, for nothing, along with an old kitchen table. [Reference 1, p.224]

Apart from his work on cables, operator theory, the Heaviside layer and asymptotic series, Heaviside also postulated (as did Lorentz) a theory of motion of electrons in a magnetic field, on which the foundation of our modern cathode-ray tube lies. The picture received and displayed on modern television relies on this theory. Heaviside expanded on the famous theory of Maxwell, and gave it a vectorial format, which is used to this day. He thereby introduced the part of the total current that is due to convection, curiously overlooked by Maxwell.

Electrical engineering and physics owe Heaviside for the concept and the word 'impedance', not only in the present theory of alternating current, but in a more expanded sense, which includes switch-on and switch-off [transient] phenomena.[8,9] This framework has not yet been completely utilised.

The foundation for the modern four-pole theory of electrical networks also resulted from Heaviside's work. In three different places in his work (the first on the 18th October 1895), Heaviside not only introduced the concept of 'negative resistance', so important for understanding radio engineering, but also derived different conclusions concerning the behaviour of cables so terminated.[6,3]

A complete treatment of a lowpass filter is encountered in Heaviside's books.[6] Discussions about this filter may be found in Lagrange's studies of a cord with noncontinuous mass distribution where the mass is localised as globules at a number of points on the cord. It is quite incomprehensible that both Lagrange and Heaviside failed to notice the similarity between the two systems, especially since Heaviside always placed such emphasis on the physical interpretation of his mathematical results. Heaviside once excluded the concept of scalar and vector potential from his considerations because he found it to be a mathematical abstraction (as opposed to electric and magnetic forces), to which one should not pay too much attention.

Closely related is Heaviside's concept of an electromagnetic wave, which in principle does not undulate, but only propagates itself. This concept leads to interesting conclusions and insights which have not yet been fully realised.

I am confident that current views in the field of electrical communication engineering, both with and without the cable (which amounts to the same thing, because the ideal radio beam is the cable), in a certain sense, have brought us to the wrong path. Simple harmonic vibration, and therefore Fourier analysis, is often singularly overexaggerated. Electrical switching phenomena, which can be followed in such an elegant manner with the modern cathode-ray oscilloscope, brings the eigenfunction to the forefront in a dramatic way. A study of these eigenfunctions yields a much more natural and penetrating analysis of the phenomena than considerations of sinusoidally excited impedances. Many questions concerning this concept exist which, as Heaviside said, 'Have still to be worried about.'

On a trip around the works a few months ago, I brought along the works of Heaviside as scientific reading. The painstaking nature of this literature caused me to speak about the work of this much forgotten recluse, who wrote his results in (at that time) a nearly unreadable form, and who richly deserved the recognition which came towards the end of his life. Meanwhile, the development of the operator method now makes it possible for modern electrical engineering to draw from the abundance of ideas entrapped by Heaviside in his works. The time is ripe for their further development. Results are still obtained which have important applications in purely mathematical fields as well as physics and electrical engineering. There is still a large and unexplored field that is worthy of the attention of our Institute.

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